This integral is an incomplete gamma function $Q(a, r_o)$. In this function a is a measure of the spread of the distribution which presumably would not vary for the experiments described earlier. As already pointed out, however, r_o will vary with the temperature of the growth chamber, and it seems plausible that the curves shown in Figures 2 and 3 are portions of a cumulative distribution function such as that given by Equation (3).

The evidence points towards a dual role for the influence of supersaturation on contact nucleation. First, the system supersaturation influences the growth rate which in turn may be reflected in the nature of the crystal face. If the face becomes rougher or softer with a change in supersaturation, a given impact will produce more particles resulting in a higher observed rate of nucleation. Second, the system supersaturation influences the fraction of the particles produced by an impact which survive. It becomes more important to distinguish between these two factors for a crystallization process where supersaturation may not be uniform throughout the equipment, as would be the case for fines dissolution or clear liquor advance.

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NOTATION

a, b = constants in the distribution function of particles generated by a crystal impact
B^o = total number of particles generated by an impact
$B_s{}^o$ = number of particles which survive to macroscopic size
$B(L)$ = the distribution function of particles generated by a crystal impact
f_S = fraction of particles which survive
L = characteristic length of crystals
$Q(a, r_o)$ = incomplete gamma function
r_o = critical length of a nucleus
R = gas constant
T = system temperature
V = molar volume of the crystalline substance
y_A = mole fraction of solute in liquid
$y_{A,S}$ = saturation mole fraction of solute at the system temperature
α = supersaturation ratio, $y_A/y_{A,S}$
$\Gamma(1 + a)$ = gamma function
σ = interfacial energy

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Oscillations on Sieve Trays

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In several recent articles Biddulph and Stephens (1974) and Biddulph (1975) have studied lateral oscillations occurring on distillation sieve trays. They have identified two types of stable oscillation which they term "full wave" and "half wave," respectively. In full-wave oscillations, the dispersion on both sides of the tray simultaneously moves towards the tray center and then back again. In half-wave oscillations, the liquid on the tray sloshes from side to side. At gas loadings intermediate between those to initiate either type of oscillation, the nature of the biphase on the tray becomes confused with peaks moving around the tray. From a tray design viewpoint, the presence of half-wave oscillations in particular is shown to be undesirable, as tray entrainment may be increased by as much as 70%.

Biddulph and Stephens (1974) have found that the experimental initiation points for both types of oscillation can be satisfactorily correlated by the group

$$B_S = \frac{V \epsilon h_f \rho_g}{g d^3 \rho_L \bar{\alpha}} \tag{1}$$

with B_S taking the value 0.5×10^{-5} for full-wave oscillations and 2.5×10^{-5} for half-wave oscillations. The cube root of the ratio of the critical values of B_S is 1.71 compared with the expected value of 2.0 based on the hypothesis that half-wave oscillations occur at double the wavelength for full-wave oscillations. No satisfactory explanation is offered for this apparent discrepancy.

However, a closer examination of available experimental evidence on the gas velocity at the initiation point for half-wave oscillations reveals that this velocity closely corresponds to the velocity at which phase inversion occurs on the tray. This can be seen from Table 1 in which reported gas velocities at the half-wave initiation point (expressed in terms of F_S, the tray superficial F factor based on active area) are compared with predicted superficial F factors for the phase inversion point.

This, therefore, suggests that the appearance of half-wave oscillations is coincident with phase inversion on the tray and that the initiation velocity for half-wave oscillations corresponds to the velocity at which the dispersion on the tray is primarily a spray (that is, to the velocity at phase inversion or the froth to spray transition). The apparent discrepancy in the critical B_S values of Biddulph and Stephens (1974) may be simply explained. Since with phase inversion the nature of the dispersion changes from liquid continuous to gas contin-

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TABLE 1. AVAILABLE DATA ON INITIATION POINT FOR HALF WAVE OSCILLATIONS

Hole diameter, cm	Tray free area, %*	Liquid flow, m^3/hr.-m weir	Experimental gas rate (F_S), ms^{-1} $(kg/m^3)^{1/2}$	Gate rate at phase inversion (F_S), ms^{-1} $(kg/m^3)^{1/2}$	Reference
0.64	11.8	14.8	2.68	2.60	Biddulph and Stephens (1974)
0.64	11.8	19.8	2.80	2.68	
0.64	11.8	23.4	2.83	2.72	
0.64	5.9	23.4	2.15	2.09	
0.30	12.8	19.8	2.38	2.45†	Biddulph (1975)
0.48	12.9	5.2	2.22	2.50	McAllister, McGinnis, and Plank (1958)
1.27	13.0	5.2	2.13	2.19	
0.64	13.1	5.2	2.36	2.44	
0.48	12.9	5.2	2.39	2.50	
0.64	6.7	5.2	1.94	2.04	
0.80	6.0	5.6	2.20	1.92	
0.80	6.0	0.9	2.05	1.92**	McAllister and Plank (1958)
0.48	5.0	7.5	2.21	1.96	Barker and Self (1962)

* Based on active or bubbling area = tower area − 2 × downcomer area.

† Estimated from data of Porter and Wong (1969) and Burgess and Robinson (1969).

** Estimated from data at lowest available liquid cross flow.

uous, there is no reason to expect the critical B_S value for full-wave oscillations which occur in the liquid continuous regime to be directly related to the critical B_S value for half-wave oscillations which occur in the gas continuous regime.

In preparing Table 1, predicted values of F_S at the phase inversion point for the particular tray geometry and liquid loading involved have been taken from the experimental measurements of Loon et al. (1973), and Pinczewski and Fell (1972). These apply to the air-water system and encompass hole diameters of 0.64, 1.27, and 1.91 cm, tray free areas of 5 to 15%, and liquid loadings of 5 to 58 m^3/hr.-m weir. Values for trays having hole diameters of 0.48 cm have been obtained by extrapolation of these data by using a corrected expression of the type proposed by Jeronimo and Sawistowski (1973). For the 0.30 cm hole diameter tray (Biddulph, 1975), an estimate of the value of F_S at phase inversion has been made from the data of Porter and Wong (1969) and Burgess and Robinson (1969). Table 1 does not include oscillation initiation velocity data for trays with hole diameters less than 0.3 cm (Biddulph, 1975; Thomas, 1964; McAllister et al., 1958; McAllister and Plank, 1958) or for trays on which the liquid cross flow is less than 5 m^3/hr.-m weir (McAllister et al., 1958), as reliable phase inversion measurements have not been made on such trays. Also not included is a recently reported oscillation initiation point of Zanelli and Del Bianco (1973) because of the absence of phase inversion data for the unusual tray geometry (very low active area) used by these authors.

The bulk of the reported velocities at the half-wave initiation point on Table 1 agree with predicted velocities at phase inversion to within the expected accuracy of the latter (estimated as ± 5% for trays with holes $\geqq$ 0.64 cm, ± 10% for remainder). However, one significant observation is that half-wave initiation velocities reported by McAllister et al., (1958) at low liquid cross flows (not included in Table 1) show an inverse dependence of initiation velocity on liquid rate in apparent conflict with the expected dependence of the related phase inversion velocity on liquid loading. It is suggested that this can be explained in terms of tray stability as discussed below.

TRAY STABILITY IN THE SPRAY REGIME

Since the appearance of half-wave oscillations is a phenomenon of the spray regime, it is possible to reexamine available tray stability criteria accounting for the particular nature of the dispersion in the spray regime. The present authors (1974) have shown that the dispersion in the spray regime is comprised of a mass of droplets undergoing trajectories in the intertray space. Crudely, the dispersion can be viewed as a fluidized bed of droplets as suggested by Andrew (1969) and treated as inviscid ($\epsilon = 0$).

By making use of the stability analysis initially performed by Hinze (1965) and corrected by Zanelli and Del Bianco (1973), it is possible to derive a criterion for wave stability and growth. Neutral stability occurs for

$$N_1 = 2\left\{1 - \frac{2(1-\alpha_p)^2}{K(1-\bar{\alpha})}\right\} \quad \text{and} \quad N_2 = 2 \qquad (2)$$

with growth for wave numbers $N_1 < N < N_2$. On most sieve trays, under normal operating conditions, both roots approximate $N = 2$ or $2\Pi h_f/l = 2$ with l, the wavelength, equal to twice the tray width for half waves.

Hence the criterion for the appearance of half-wave oscillations on a tray in the spray regime becomes

$$h_f \approx 0.6 \times (\text{effective tray width}) \qquad (3)$$

The assumptions made in the Hinze stability analysis necessarily mean that the criterion proposed in Equation (3) is inexact. Nonetheless, it would appear that half-wave oscillatory behavior is to be expected in experimental scale columns, where the ratio of dispersion height to tray width is appreciable. This is in accord with qualitative observations in the present authors' laboratory, where it has been found that oscillatory behavior is associated with high froth or dispersion heights. Moreover, it explains the apparently high velocities observed by McAllister et al. (1958) for half-wave initiation at low liquid flow rates; although their trays would appear to have been operating well into the spray regime, an observable oscillation was not apparent until much higher velocities, where the height of the dispersion had become appreciable.

The analysis of Hinze (1969) also allows for the possibility of stable oscillations in the fully developed spray regime with wavelengths equal to integral multiples of the effective tray width. These will occur with increase in tray diameter as the ratio of dispersion height to effective tray width becomes successively smaller. They have not as yet been reported on experimental scale trays, doubtless because of the small diameter of most such trays. Accentuation of such oscillations by maldistribution of the gas between successive trays as in the half-wave case will not be so severe, and the gross effect of these oscillations on tray entrainment is not expected to be as pronounced as that observed by Biddulph and Stephens (1974). At the small ratios of dispersion height to tray width prevailing on industrial sieve trays operated in the spray regime, the possibility of oscillations is not considered to present a major obstacle to the design and operation of such trays.

DISCUSSION

In their analysis of the likelihood of oscillatory behavior on sieve trays, Biddulph and Stephens (1974) have equated the eddy viscosity used in their dimensionless group (B_S) to the liquid eddy diffusivity measured by a tracer technique. The validity of this assumption is particularly questionable in the spray regime in which the continuous phase is the gas phase. Indeed, when a realistic gas phase eddy viscosity ($\epsilon \rightarrow 0$) is used in Biddulph and Stephens' criterion for half-wave oscillations, it is predicted that the experimentally observed half-wave oscillations on small scale trays will not occur.

It is, therefore, of interest to reexamine the half-wave criterion ($B_S = 2.5 \times 10^{-5}$) they have proposed for the case of operation in the spray regime. For a given tray, the eddy viscosity (diffusivity) term (ϵ) in B_S is experimentally found to scale approximately linearly with froth height, and the criterion B_S can thus be rewritten as

$$B_S \propto \frac{V}{h_l} \cdot \left(\frac{h_f}{d}\right)^3 = 2.5 \times 10^{-5} \qquad (4)$$

Since at the point of phase inversion $V \propto h_l$ (Pinczewski and Fell, 1972), the criterion proposed in Equation (4) is fortuitously the same as that given directly by Hinze's analysis. However, the present treatment in which the eddy viscosity of the dispersion in the spray regime is correctly accounted for is considered preferable. It is lent support by the finding of Zanelli and Del Bianco (1973) that the frequency of half-wave oscillations closely agrees with that expected for an inviscid dispersion.

The above comments do not preclude the use of the correlation of Biddulph and Stephens (1974) to analyze for the likelihood of full wave oscillations occurring on a tray operated exclusively in the froth regime. However, as these authors have suggested (1974), full-wave oscillations are unlikely to occur from hydrodynamic considerations alone in columns of diameter greater than 1.0 m operated at atmospheric pressure and above.

CONCLUSION

The violent half-wave oscillations previously observed on experimental scale sieve trays are associated with the spray regime operation of such trays and would appear to be an artifact of the high dispersion height to tray width ratio then prevailing. They will occur from the phase inversion point if the froth height to tray width ratio is sufficient. On the basis of Hinze's stability analysis, they are not expected to occur on industrial trays, since on such trays the tray width is very much greater than the tray height.

There remains the possibility of less severe full-wave oscillations on industrial scale trays. For trays operating in the froth regime, the analysis of Biddulph and Stephens (1974) can be used to predict the likelihood of such oscillations. This analysis is no longer valid for trays operating in the spray regime but may be replaced by a corrected Hinze analysis in which the dispersion is treated as inviscid.

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NOTATION

B_S = dimensionless group based on superficial vapor velocities
d = column diameter, m
F_S = superficial F factor based on active area $(V\sqrt{\rho_g})$ $\text{ms}^{-1}(\text{kg/m}^3)^{1/2}$
h_f = mean froth height, m
h_L = clear liquid head, m
K = constant proportional to discharge coefficient
l = wavelength, m
N = wave number = $2\Pi h_f/l$
V = superficial vapor velocity, ms^{-1}

Greek Letters

α_p = 1 − fractional free area of sieve plate
$\bar{\alpha}$ = mean liquid fraction by volume
ϵ = eddy kinematic viscosity, m^2s^{-1}
ρ_g = gas density, kgm^{-3}
ρ_L = liquid density, kgm^{-3}

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